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Investigation of student understanding of the concept of acceleration in one dimension

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This paper describes a systematic investigation of the understanding of the concept of acceleration among students enrolled in a variety of introductory physics courses at the University of Washington. The criterion for assessing understanding of a kinematical concept is the ability to apply it successfully in interpreting simple motions of real objects. The main thrust of this study has been on the qualitative understanding of acceleration as the ratio $\Delta v/\Delta t$. The primary data source has been the individual demonstration interview in which students are asked specific questions about simple motions they observe. Results are reported for the success of different student populations in comparing accelerations for two simultaneous motions. Failure to make a proper comparison was due to various conceptual difficulties which are identified and described. Some implications for instruction are briefly discussed.

I. INTRODUCTION

The Physics Education Group at the University of Washington has been engaged for several years in a systematic study of the ways in which students in introductory college physics courses think about motion. The degree of difficulty of the courses ranges from compensatory (for academically disadvantaged students) to calculus based (for physics, engineering, and mathematics majors). This article, the second of two devoted to the kinematical concepts, reports on the ability of students to apply the concept of acceleration in interpreting simple motions of real objects. An earlier paper discusses student understanding of velocity.\(^1\)

The motivation and methods for the research, the criterion selected to assess understanding, and the student populations involved in the investigation have been discussed in the earlier article.\(^1\) This paper begins with a summary of that discussion and then describes the means used to assess student understanding of the concept of acceleration. The results are analyzed, interpreted, and examined for their instructional implications. We conclude with a brief discussion of the usefulness of this type of investigation.

II. DESCRIPTION OF RESEARCH

In this study we have adopted as a measure of understanding of a kinematical concept the degree to which an individual successfully applies that concept to the interpretation of simple motions of real objects. The main thrust of this study has been on the qualitative understanding of acceleration as the ratio $\Delta v/\Delta t$. The primary source of data has been the individual demonstration interview in which students are asked specific questions about motions they observe. These interviews are administered to students from a wide variety of introductory physics courses before and after instruction in kinematics. Information obtained in this way has been supplemented by results from course examinations.

The tasks the students are asked to perform during the interviews usually involve comparisons of the accelerations of two motions. In these tasks two identical steel balls roll along a pair of aluminum $U$ channels inclined at the same angle to the horizontal. The accelerations of the balls can be varied by using channels of different widths as shown in Fig. 1. Thus prior knowledge about the dependence of acceleration on slope yields no clues for making correct comparisons. A mechanism for releasing the balls automatically insures that the motions are reproducible.

The interviews are conducted according to a standard questioning format but at any point the interviewer may choose to probe more deeply into a student's understanding by extending the discussion. The interviews, lasting from 20 to 30 min each, are audio taped and occasionally video taped. Often questioning about acceleration is preceded by administration of a speed comparison task.\(^1\)

The students who participated in this study on acceleration were enrolled in several different types of introductory physics courses. One of these was a special three-quarter basic physics sequence for academically disadvantaged students (AD).\(^2\) This course was limited to participants in the Educational Opportunity Program (EOP) who had expressed an interest in a health science or other science-related profession. Other courses involved in the investigation included a special self-paced section of a noncalculus general physics course (GPS), a regular lecture section of this same course (GPL), a regular lecture section of a calculus physics course (CP), and a class of in-service elementary school teachers (IT).

III. SOURCES OF DATA

A. Replication and extension of Piagetian acceleration task

To establish a benchmark for our subsequent work, we replicated four of the motion tasks developed by Jean Piaget.\(^3\) Three of the tasks involve uniform speed while the fourth deals with uniform acceleration. The first three tasks have been described previously.\(^1\) The discussion here will be confined to the fourth task, which is the only Piagetian task dealing with nonuniform motion. Piaget considered its successful resolution evidence for formal thought.

The Piagetian acceleration task calls for qualitative predictions of the relationship between distance traveled and elapsed time for a ball rolling down an incline. During an interview the subject is asked to compare the distances
covered by the ball in equal successive time intervals or to compare the time intervals necessary for the ball to traverse equal distances. When we administered this task to EOP students who had never studied motion, they experienced very little difficulty with it. Our results with college students, therefore, differed from those Piaget obtained in his work with children, among whom he found a wide range of performance on this task.

Successful completion of the Piagetian acceleration task does not require consideration of a ratio. A primitive intuition of acceleration as “speeding up” appeared to be adequate. Since this task seemed to present no challenge to the students interviewed, we tried modifying it to make it more quantitative. In the apparatus shown in Fig. 2, a ball is released from rest and rolls down an inclined aluminum U channel. First, its motion over the first 50 cm is timed with a stop clock. Then, the student is asked to predict approximately how much time it would take for the ball to cover 100 cm from its starting point. About half of the students in the EOP class predicted correctly that it would take less than twice as long to reach the 100-cm mark. When we presented this modified task to students in the noncalculus general physics course, we found that they were almost all successful in making the correct prediction. Thus we found that most college students in our study could deal successfully with a semiquantitative extension of the Piagetian acceleration task.

B. Exploratory acceleration comparison task

1. Description of task

A satisfactory solution of the Piagetian acceleration task does not require understanding of the concept of acceleration at the level expected in the study of physics. As the first step in the development of a more appropriate task, we conducted exploratory interviews on acceleration with the demonstration setup from Speed Comparison Task 1, which we had used to investigate student understanding of velocity. The apparatus is illustrated in Fig. 3 and the motion the students observed in Fig. 4. (The graph was not used in the interview.) In this task ball A travels at constant velocity with no acceleration. Ball B, which has a greater initial velocity, is released later from a point behind ball A. Since ball B slows down as it travels up the incline, it has an acceleration different from zero. The students are asked to compare the accelerations of the two balls.

We decided to work first with students with the weakest backgrounds in physics and hence interviewed members of the academically disadvantaged class. During these interviews, which took place after the study of acceleration in the first year the course was offered, the students frequently referred to acceleration as a “change in velocity over time.” We attempted to delve more deeply into what these words meant to the students by questioning them about the motions they observed.

2. Example of student difficulty

The following excerpt from one of these interviews is typical of responses made by about half of the class. (The letter “I” represents the investigator and the letter “S” the student.)

I: Do these two balls ever have the same acceleration?
Both balls are released.
S: Wouldn’t they have the same acceleration at the point they have the same velocity?
I: Why do you think that would be true?
S: Because your acceleration is that delta v over delta t. And at the point where you have the same velocity, you have the same delta t and the same delta v.

A successful comparison in this task does not require a quantitative understanding of acceleration. It does require that an individual be able to think of $\Delta v$ as a difference between two values of the velocity. The student quoted above was unable to make the necessary distinction between the concepts of velocity and change of velocity.

Students who recognized that acceleration implies change in velocity were able to complete this task correctly. Further probing, however, indicated that in many cases the word “over” often used by students in defining acceleration did not necessarily refer to the relationship between the numerator and the denominator of the fraction $\Delta v/\Delta t$. For
these students "over" was equivalent to "during." They realized that the concept of acceleration includes the idea of a change in velocity but did not recognize that it also incorporates in an explicit manner the idea of a corresponding time interval during which this change takes place. From these interviews it became apparent that what was needed was a task which would require for successful completion an understanding of the role of $\Delta t$ as well as $\Delta v$.

C. Acceleration Comparison Task 1

1. Description of task

In this task (Fig. 5), students observe the motions of two steel balls which roll down straight aluminum $U$ channels which are placed side by side and inclined at the same angle to the horizontal. The channels are of different widths. Both balls start from rest and reach the same final velocity at the end of the incline just as they simultaneously enter a tunnel at the bottom. They are not released at the same point or the same time and do not travel equal distances. Ball $A$ is released first from a point several centimeters behind ball $B$. After rolling a few centimeters, ball $A$ strikes the lever of a microswitch which in turn releases ball $B$. The two motions are illustrated in the accompanying graphs (Fig. 6 and Fig. 7), neither of which is used in the interviews. As can be seen from the graphs, the balls have the same average velocity and the same final velocity. However, ball $B$, which rolls on the narrower channel, reaches that velocity in a shorter period of time than ball $A$ and has an acceleration about 15% greater than that of ball $A$.

The balls are rolled separately at first and the fact that each has an acceleration is established. During the course of the interview, the student is asked, "Do these two balls have the same or different accelerations?" Two procedures for arriving at the correct conclusion that ball $B$ has a greater acceleration than ball $A$ follow immediately from the definition of acceleration as $\Delta v/\Delta t$. The student must recognize either that (a) since ball $A$ is already moving when ball $B$ is released, the change in velocity for ball $B$ is greater than the change in velocity for ball $A$ between the instant ball $B$ is released and the instant both balls enter the tunnel; or that (b) since both balls start from rest and reach the same final velocity, ball $B$, which is released after ball $A$, makes this change in a shorter period of time. Although entirely qualitative, both methods require explicit consideration of both $\Delta v$ and $\Delta t$ in determining the acceleration. Procedures (a) and (b) are designated as Procedures 9 and 10, respectively, in Sec. III C3, in which the various procedures used by students on Acceleration Comparison Task 1 are summarized.

To encourage students to concentrate on the main conceptual issue rather than on subsidiary experimental details, specific guidance is provided. The interviewer explains that to make the comparison it is unnecessary to identify the cause of the acceleration or to determine whether or not the balls, the channels, or the slopes are the same. The comparison of accelerations is to be made strictly on the basis of the motions observed. It is pointed out that ball $A$ is released first and rolls for a short time before hitting the switch that releases ball $B$. If a student does not notice that the balls enter the tunnel at the same time or does not spontaneously compare final speeds, the interviewer asks questions that serve to direct attention to the arrival of the balls at the tunnel. Thus the students are assisted in making the observations necessary for comparing the accelerations. It remains for them to combine this information in a manner that permits successful resolution of the task.

2. Examples of student difficulties

Acceleration Comparison Task 1 was administered both before and after instruction in more than 200 individual demonstration interviews with introductory physics students. We found that all the responses could be organized into ten categories, each characterized by the procedure used to compare accelerations. Two of the procedures are directly based on the definition of acceleration and have already been described. The other eight, each identified by a number, are discussed below. All but the first are illustrated by excerpts from interviews. These excerpts, which have been edited to eliminate irrelevant dialog, are typical of responses of students who used a particular procedure. When pre- and postcourse excerpts appear together, they are from interviews with the same student. This juxtaposition is intended to illustrate the persistent nature of some of the mistaken conceptions with which students often begin physics courses.

Procedure 1:

One misconception that is often true but is not true in this case can severely limit a student's ability to deal with Ac-
acceleration Comparison Task 1. The belief that the accelerations of the two balls must be the same because they are on the same incline was initially expressed by a large percentage of the students, especially those who had studied physics. Usually we could dissuade students from using a procedure based on slope by questioning about various parts of the motion or by simply pointing out that the channels were not identical and suggesting that dynamical considerations be ignored. Since this procedure was almost always abandoned early in the interview, it is not illustrated by an excerpt.

Procedure 2:

I: Let's see whether we can decide whether they have the same acceleration or different accelerations. (Balls are released.)
S: Ball A was a little bit faster. (It was a fraction of an inch ahead of ball B at the end.)
I: (After repeated demonstrations.) Ball A was released first. Then, ball B was released, and down there they had the same speed. Does that tell you anything about their accelerations?
S: I think they have the same acceleration. They ended up there at the same point.

The dialog above illustrates the belief that when the two balls reach the same position they have the same acceleration. The excerpt is taken from a postcourse interview with a student from the academically disadvantaged class. In claiming that when the two balls are at the same point they have the same acceleration, this student used a position criterion to compare accelerations. This procedure is reminiscent of a similar one used by some students to compare velocities. Sometimes during the interviews students would state that the ball that was ahead would have to have the greater acceleration.

Many of the difficulties students had with Acceleration Comparison Task 1 seemed to be due to confusion between the concepts of velocity and acceleration. In referring to a confusion, we are using this word in a restricted sense. It is not intended to indicate mistaking of one fully developed concept for another but rather to characterize thinking in terms of nondifferentiated protoconcepts. The next four sets of interview excerpts demonstrate a failure to discriminate between the concepts of velocity and acceleration. In the first two sets, the concept of velocity is undifferentiated and largely intuitive. The judgment made by each student is almost entirely perceptual. In the second two sets, the procedures seem to be based on a sense of average velocity.

Procedure 3:

Precourse
I: How would we compare their accelerations? (Balls are released.)
S: Towards the end, they were going about the same rate. (Demonstration is repeated.) They would eventually reach the same acceleration.
I: How do you know?
S: They had the same speed at the bottom, approximately.

Postcourse
I: What does the word “acceleration” mean?
S: It's the change in velocity over a certain period.
I: Do you think, when they go in the tunnel, they have the same velocity?
S: They seemed to.

I: And how do they start out?
S: It seems one would have a greater velocity, initially.
I: Can you use that to compare their accelerations?
S: They had to be equal.

The pair of excerpts above illustrates a procedure in which students attempted to compare accelerations on the basis of final speeds alone. The student quoted is from the self-paced section of the noncalculus general physics course. The definition given of acceleration as “the change in velocity over a certain period” might well have been accepted as indicative of understanding if discussion had ended with that statement. As a matter of fact, the student completed the requirements of the course quite satisfactorily and received a top grade. As his subsequent remarks indicate, however, he thinks of acceleration as a change in velocity that occurs as time passes. His use of the word “over” does not imply division.

Procedure 4:

Precourse
I: If we're comparing two balls and we decide one has a bigger acceleration, what would that mean?
S: It speeds up in a shorter time.
I: Let's see if we can decide which has a larger acceleration or whether they're the same. (Balls are released.)
S: They came together. So A had a larger acceleration. It had to be because it had to catch up with B which was ahead of it.

Postcourse
I: We would like to decide whether they have the same or different accelerations. (Balls are released.)
S: Ball A would have the greater acceleration because in order for A to catch up it would have to go faster.

These excerpts display a procedure that was frequently used to compare accelerations. It is based on the belief that catching up (in the sense of gaining on) means having a greater acceleration. Catching up, of course, merely indicates nonzero relative velocity. In Acceleration Comparison Task 1, ball A catches up to ball B; yet ball A has a smaller acceleration. The student quoted above completed the calculus physics course in the top half of the class according to the final course grades as did all the other students quoted in the rest of this section.

Confusion between the concepts of velocity and acceleration was also demonstrated by students who formed an over-all impression of the velocities of the balls over the entire motion. These students never focused their attention on separate parts. The two sets of interviews that follow are illustrative of this way of proceeding.

Procedure 5:

Precourse
I: Let's see whether these two balls have the same acceleration or different. (Balls are released.)
S: It looks like ball A would have a faster acceleration.
I: How are you thinking?
S: Ball A covered more distance in the same amount of time.

Postcourse
I: In terms of speed and things like that, how would...
to the same speed that \( B \) did and \( B \) had a shorter distance.

This excerpt from a postcourse interview is reminiscent of Galileo's problem of determining which definition of acceleration to choose—a change in velocity per unit distance or a change in velocity per unit time. Here, a calculus physics student did not discriminate between these two possibilities and associated the change in velocity of the balls with the distance they traveled rather than with the elapsed time. Although this student arrived at the correct solution to the task, he did not consider a time interval in his response.

**Procedure 8:**
- **I:** Why do you think that was the case?
- **S:** Because ball \( A \) gained position on ball \( B \) at a greater rate on the upper part of the track than it did on the bottom part, which I would interpret as the velocities coming closer together \ldots \ Ball \( B \) would have a greater acceleration.

This student from the calculus physics course also made the correct choice in this postcourse interview. He argued that since ball \( B \) had a considerably smaller velocity than ball \( A \) at the beginning, but nearly the same velocity later on, ball \( B \) must have had a greater acceleration. This particular response may or may not have represented a conceptual difficulty. It is included here because the student did not make explicit reference to the time interval or compare the accelerations as ratios.

### 3. Summary of procedures

The various procedures used by students as they attempted to compare accelerations on Acceleration Comparison Task 1 are listed in the chart in Table I. The first six procedures in the chart lead to an incorrect solution. Although the seventh yields the right response, the type of reasoning involved is inadequate. The eighth procedure results in the correct judgment of which ball has the greater acceleration but does not explicitly involve the time interval. We have placed this procedure at a lower level than Procedures 9 and 10 in which the corresponding time interval is identified. Some students used more than one procedure during an interview. The prevalence of the different types of inadequate procedures used by students is discussed in Sec. IV.

### D. Acceleration Comparison Task 2

#### 1. Description of task

Acceleration Comparison Task 1 was used to assess qualitative understanding of the ratio \( \Delta v/\Delta t \). In order to examine quantitative understanding of the concept of acceleration, we developed Acceleration Comparison Task 2. This is a paper and pencil task for which no apparatus is used. The task requires a comparison of the accelerations of two balls rolling down inclined tracks onto sections of level track. The solution depends on the ability to interpret acceleration as a ratio \( (\Delta v/\Delta t) \) that can be constructed from measurements of distances and times. Statement of the problem is accompanied by directions intended to focus the attention of the students on the interpretation of acceleration as a ratio rather than on the use of kinematical for-
Table I. Summary of procedures used by students on Acceleration Comparison Task 1.

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Interpretation of procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Balls have the same acceleration because the slopes are the same.</td>
<td>Nonkinematical approach</td>
</tr>
<tr>
<td>2. Balls have the same or different accelerations depending on their</td>
<td>Confusion between position and acceleration</td>
</tr>
<tr>
<td>relative final positions.</td>
<td></td>
</tr>
<tr>
<td>3. Balls have the same acceleration because their final speeds are</td>
<td></td>
</tr>
<tr>
<td>the same.</td>
<td></td>
</tr>
<tr>
<td>4. Ball A has a greater acceleration because it is catching up to</td>
<td>Confusion between velocity and acceleration</td>
</tr>
<tr>
<td>(gaining on) ball B.</td>
<td></td>
</tr>
<tr>
<td>5. Ball A has a greater acceleration because it covers a greater</td>
<td></td>
</tr>
<tr>
<td>distance than ball B in the same time.</td>
<td></td>
</tr>
<tr>
<td>6. Balls may have the same acceleration because ball A covers a</td>
<td></td>
</tr>
<tr>
<td>greater distance than ball B in a longer time.</td>
<td></td>
</tr>
<tr>
<td>7. Ball B has a greater acceleration because its velocity changes</td>
<td>Discrimination between velocity and change in velocity</td>
</tr>
<tr>
<td>by the same amount as the velocity of ball A but in a shorter distance.</td>
<td>but neglect of corresponding time interval</td>
</tr>
<tr>
<td>8. Ball B has a greater acceleration because its velocity catches up</td>
<td></td>
</tr>
<tr>
<td>to the velocity of ball A and thus changes by a greater amount.</td>
<td></td>
</tr>
<tr>
<td>9. Ball B has a greater acceleration because its velocity changes by</td>
<td>Qualitative understanding of acceleration as the ratio</td>
</tr>
<tr>
<td>a greater amount than the velocity of ball A in the same time.</td>
<td>( \Delta v/\Delta t )</td>
</tr>
<tr>
<td>10. Ball B has a greater acceleration because its velocity changes by</td>
<td></td>
</tr>
<tr>
<td>the same amount as the velocity of ball A in a shorter time.</td>
<td></td>
</tr>
</tbody>
</table>

mulas. The task may be administered either during an interview or on a written course examination.

Acceleration Comparison Task 2 is reproduced in Fig. 8 in the same form in which it is presented to the students. The data given for the level portions of the track may be used to determine the velocities of the balls at the bottom of the inclined channels. The initial velocity of both balls is zero. Thus the change in velocity for each of the balls can be readily found and used with the corresponding time intervals to calculate the accelerations. The information provided about the length of the incline is irrelevant to completion of the task. A correct solution of the problem yields a value for the acceleration of ball A of 9.0 cm/sec² and of ball B of 7.5 cm/sec². Thus ball A has the greater acceleration.

Success on this task is defined to mean that a student used a valid procedure for comparing accelerations, besides substituting in a kinematical formula, such as \( \Delta s = \frac{1}{2}a(\Delta t)^2 \) or \( v^2 - v_0^2 = 2a(\Delta s) \). Conceptual understanding of acceleration is not required in order to manipulate the formulas to give the desired answer. If the student ignored the instantaneous velocity or confused instantaneous with average velocity, the student was considered to have failed the task. When this task appeared as a question on course examinations, an unintelligible or omitted response was scored as a failure.

2. Example of student difficulty

Acceleration Comparison Task 2 was used only after the students had studied kinematics. It was administered as a question on course examinations to about 200 students in three introductory physics courses (AD, GPL, and CP). It was also presented during interviews to 23 students from the self-paced noncalculus general physics course (GPS). During the individual demonstration interviews, the students were encouraged to describe their thinking as they attempted to solve the problem. These interviews provided us with a more complete description of conceptual difficulties than we could have obtained from the examinations alone.

Following is an excerpt from a postcourse interview on this task. This student's response was representative of the performance of most of the students who failed to complete the task correctly.

I: Using these numbers here, could you determine which ball has a greater acceleration on the sloping part?

S: (Writes \( v = \frac{d}{t} \) and \( a = \frac{v}{t} \). Calculates 40.5/3.0 = 13.5 and 60.0/4.0 = 15, and then divides 13.5/3 = 4.5 and 15/4 = 3.75.) Ball A would have a greater acceleration.

I: How did you figure that out?

S: Well, first I figured out the average velocity for this part (indicates bottom of sloping section). And then I got that number, which is 13.5 and then divided by the 3 sec. (Similar reasoning for ball B.)

D. Trowbridge and L. C. McDermott 247

Fig. 8. Acceleration Comparison Task 2. Written question presented without apparatus on course examinations and in some interviews: "Two balls roll down sloping sections of track (not necessarily the same slope) and onto level sections where they have uniform motion. Times and distances are measured and shown in the diagram. According to the information given, which ball has the greater acceleration on its sloping section? CAUTION: No credit will be given for the use of the formula, \( \Delta s = \frac{1}{2}a(\Delta t)^2 \). Use instead the definition of acceleration."
I: So you conclude that...?
S: A has a greater acceleration.

Although the student made the correct choice, the method is incorrect. The difficulty illustrated proved to be widespread among all student populations included in this study. Students would often write \( v = \frac{d}{t} \) and \( a = \frac{v}{t} \), proceed to calculate the average velocity on the incline, and then use this value, instead of the instantaneous velocity at the end of the incline, to find the acceleration. This confusion made successful solution of the problem impossible. Moreover, discrimination between instantaneous and average velocity is essential for a numerical interpretation of \( \Delta v \), which in turn is crucial for an understanding of acceleration.

E. Exploratory task for a special case of uniform acceleration

1. Description of task

Although the main thrust of this investigation has been on the interpretation of acceleration as a ratio, we extended some of the interviews to explore student understanding of a special case of uniform acceleration. The kinematical description of a projectile is a problem included in almost every introductory physics course. It has long been recognized as causing particular difficulty for students. The existence of an acceleration that remains constant in magnitude and direction while the velocity undergoes constant change often is not readily accepted. We adapted one of the interview demonstrations used in the speed comparison tasks (Fig. 3) to investigate how students apply the concept of acceleration in a situation very similar to projectile motion.

Ball B is launched with some initial velocity, travels up a gentle incline, reverses direction, and returns to its starting point. Three parts of the motion are identified: (1) the period in which the ball is traveling up the incline, (2) the instant at which the ball is at the top of the incline, and (3) the period in which the ball is traveling down the incline. The student is asked to describe the acceleration throughout the entire motion. If the student does not comment on part (2), the interviewer asks a direct question about whether there is an acceleration at the instant the ball is at the top of the incline.

2. Examples of student difficulties

Two excerpts taken from postcourse interviews follow, both of which illustrate the tendency to think of the acceleration as zero at the top of the incline. The first involves a student from the academically disadvantaged class and the second a student from the self-paced general physics course.

S1: 1: What is the acceleration there right at the top?
S: Zero. Because it's stopped. It's not moving; it's turning around.
S2: 1: Can you say anything about the acceleration at that instant?
S: The acceleration at the instant the ball comes to rest is zero, because it has zero velocity, and there's no change in time. Acceleration would be identifiable as change in velocity over change in time.

In the following dialog from a postcourse interview, a student from the calculus physics course discusses the change in the sign of the acceleration. He claims that the sign of the acceleration changes with the sign of the velocity.

I: Let's imagine that we computed some number for the acceleration as it's going up the track, and we did the same thing as it's coming down the track. Do you have any feeling for how those numbers would compare with each other?
S: The acceleration going up the track would be negative, and equal in magnitude to the positive acceleration coming down the track.

Some students stated that the acceleration was zero at the top of the incline for a different reason from the one illustrated in the first two excerpts. They expressed the belief that when the direction of motion of the ball changed, the direction of the acceleration changed and therefore had to pass through zero.

These same types of responses occurred among all student populations. In a sample of 39 students from the calculus physics course, 64% were unsuccessful with this task on precourse interviews, while 36% still responded incorrectly on postcourse interviews. Thus, even after instruction, about one-third of the students continued to have difficulty with the idea of an acceleration that remains constant when the velocity does not.

IV. ANALYSIS OF RESULTS

We may present the data from the administration of Acceleration Comparison Task 1 in a semiquantitative form similar to the one used for velocity. A detailed discussion of the quantification of the data may be found in the dissertation describing this research. Numerical scoring of a student's performance on the task was based on an assessment of over-all quality according to the criteria described below.

The performance of students on Acceleration Comparison Task 1 was measured on a three point scale of 0, 1, or 2. A score of "2" meant that the student, with limited guidance by the interviewer, was able to make a qualitative comparison of ratios. A score of "1" indicated that the student compared instantaneous velocities at different times but never referred to the specific time intervals during which the changes in velocity occurred. When after a considerable amount of questioning by the interviewer, the student still had made no attempt to use instantaneous velocities at different times for comparing accelerations, a score of "0" was assigned. The specific scoring criteria are shown in Table II.

The various categories of procedures listed in Table I were established after scoring of the interviews had been completed. Often during an interview a student would attempt to use a particular procedure, find it unsatisfactory, and then attempt to reason in some other way. Thus a number of different strategies representing varying degrees of commitment and resulting in varying degrees of success were invoked by a student during an interview. The procedures used by a student did not determine the score. Rather, the score reflected the quality of the student's over-all performance, especially his final analysis. Students who received a "0" usually relied on procedures near the beginning of the chart in Table I; students who received a "1"
Table II. Scoring criteria for Acceleration Comparison Task 1.

<table>
<thead>
<tr>
<th>Score</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student makes little or no use of instantaneous velocities at different times. Student persists throughout the interview in comparing motions in terms of the slope of the incline, the final relative positions, the final speeds, or the phenomenon of catching up.</td>
</tr>
<tr>
<td>1</td>
<td>Student compares instantaneous velocities at different times. Student may also compare changes in velocity but does not make explicit use of the time interval.</td>
</tr>
<tr>
<td>2</td>
<td>Student considers the ratio ( \Delta v/\Delta t ), compares ( \Delta v )'s for equal ( \Delta t ), or compares ( \Delta t )'s for equal ( \Delta v ). Student may display inadequate procedures during interview but successfully resolves difficulties with limited, guided questioning by the interviewer.</td>
</tr>
</tbody>
</table>

Table III. Results for Acceleration Comparison Task 1. Percentages and numbers (n) of students in each group who received scores of 0, 1, or 2...

<table>
<thead>
<tr>
<th></th>
<th>In-service teachers</th>
<th>Academically disadvantaged</th>
<th>General physics (self-paced)</th>
<th>General physics (lecture)</th>
<th>Calculus physics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(IT)</td>
<td>(AD)</td>
<td>(GPS)</td>
<td>(GPL)</td>
<td>(CP)</td>
</tr>
<tr>
<td>Precourse interviews</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>17</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>(13)</td>
<td>(17)</td>
<td>(1)</td>
<td>(13)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>7</td>
<td>(0)</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>total</td>
<td>35</td>
<td>25</td>
<td>35</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>(7)</td>
<td>(5)</td>
<td>(0)</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(15)</td>
<td>(8)</td>
<td>(20)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(20)</td>
<td>(23)</td>
<td>(28)</td>
<td></td>
</tr>
</tbody>
</table>

A. Performance on Acceleration Comparison Task 1

Acceleration Comparison Task 1 was included in pre-course interviews conducted with individuals from various introductory courses. The groups represented were in-service elementary school teachers (IT), academically disadvantaged students (AD), and calculus physics students (CP). Since the noncalculus general physics students in the self-paced course (GPS) were presented with a different form of Acceleration Comparison Task 1, data from their pre-course interviews has not been presented in this paper. The noncalculus general physics students in the lecture course (GPL) were not given pre-course interviews.

Acceleration Comparison Task 1 was also used on post-course interviews. Generally we attempted to interview the same students before and after instruction but this was not always the case. Some students who had participated in pre-course interviews did not complete the course in which they were enrolled. The in-service teachers did not receive any instruction in kinematics and therefore did not participate in postcourse interviews. Students in both the self-paced section and lecture section of the noncalculus general physics course participated in post-course interviews.

The percentages of each group of students who received each of the scores 0, 1, or 2 on Acceleration Comparison Task 1 on pre- and postcourse interviews are reported in Table III. The numbers in parentheses are numbers of students.

B. Comparison between pre- and postinstructional performance

The results from Acceleration Comparison Task 1 on both pre- and postcourse interviews for different populations of students are presented in Figs. 9 and 10. Figure 9 presents data comparing the percentages of students who received a score of at least 1 on pre- and postcourse interviews. Figure 10 presents data comparing the percentages of students who received a score of 2 on pre- and postcourse interviews. The percentages are displayed with limits of one standard deviation. We arbitrarily assigned a range of 10% to the AD group, none of whom was successful on precourse interviews. This range was chosen to be commensurate with

![Graph](image)

Fig. 9. Percentages of students in each group who received a score of at least 1 on Acceleration Comparison Task 1. Results from pre- and postcourse interviews are shown by solid and dashed lines, respectively. The uncertainties displayed are one standard deviation.


D. Trowbridge and L. C. McDermott 249
that of the IT group, which had a similar sample size and performed at about the same level. For precourse results, the data are plotted with solid lines. For postcourse results, the data are plotted with dashed lines.

Prior to instruction, between 5% and 40% of the students received a score of at least 1 on Acceleration Comparison Task 1. (See Fig. 9.) These students were able to discriminate between velocity and acceleration to the extent of being able to identify and compare instantaneous velocities at different times. As might be expected, the academically disadvantaged course had the lowest percentage of students (5%) able to meet this criterion before instruction. Approximately 40% of the calculus physics students met this criterion. In all introductory level populations studied, at least three-fifths of the students confused the concepts of velocity and acceleration on precourse interviews.

After instruction, between 35% and 70% of the students received a score of at least 1 on Acceleration Comparison Task 1. (See Fig. 9.) The percentage from the academically disadvantaged class had risen to 65%. This was comparable to the postinstructional performance of the calculus physics students. The percentages for both these groups exceeded those for the noncalculus physics students after instruction.

In the introductory level populations studied, overall about one-third of the students confused the concepts of velocity and acceleration on postcourse interviews.

Prior to instruction, between 0% and 20% of the students received a score of 2 on Acceleration Comparison Task 1. (See Fig. 10.) These students used a ratio, at least qualitatively, in comparing accelerations. No one in the class for academically disadvantaged students was successful. Approximately 20% of the calculus physics students met this criterion before instruction. In all introductory level populations studied, at least four-fifths of the students did not use ratios to compare accelerations in precourse interviews.

After instruction, between 20% and 40% of the students received a score of 2 on Acceleration Comparison Task 1. (See Fig. 10.) The percentage for the academically disadvantaged class had risen to 40%. This was comparable to the postinstructional performance of the calculus physics students. The percentages for both these groups exceeded those for the noncalculus physics students after instruction.

C. Prevalence of inadequate procedures on Acceleration Comparison Task 1

A sample of students from widely different backgrounds was selected in order to estimate the relative frequency of occurrence of the various types of inadequate procedures. Procedure 8 was included in this survey because, although it is adequate for the task, there is no explicit consideration of the time interval. The sample consisted of 35 students, 28 students from the calculus physics course and 7 from the academically disadvantaged class.

An independent investigator listened to 70 audio tapes taken from pre- and postcourse interviews with these students. The investigator tabulated instances of use of each procedure without knowledge of the identity of the students, the courses in which they were enrolled, or whether the dialogues were from pre- or postcourse interviews. Many students tried several different procedures during a single interview.

Procedure 1 was used initially by about 30% of the students. Since, however, it has a nonkinematical basis and was almost always readily abandoned, the rest of the discussion in this section will be confined to the prevalence of Procedures 2–8. Together these were used 42 times on precourse interviews and 28 times on postcourse interviews. The percentages of the total number of occurrences (%N) of the Procedures 2–8 are tabulated along with number of occurrences (n) for each of these procedures in the chart in Table IV.

Aside from use of an argument based on slopes, the most common incorrect procedure on precourse interviews was deciding that ball A had the greater acceleration because it was catching up to ball B. This type of confusion between velocity and acceleration accounted for about 40% of the errors in this sample of students. Another 20% of the errors were due to judgments based on final speeds alone.

On postcourse interviews, procedures based on catching up or final speed again were the most common, each accounting for about 20% of the errors. After instruction there seemed to be a shift in the distribution towards the more advanced procedures. It is our impression that the general pattern for the relative prevalence of incorrect procedures displayed by this sample of students resembles that for the entire group of students included in the study.

D. Effect of complicating factors

There are several factors inherent in this type of study that must be examined for influence on the results. Among
Table IV. Frequencies of occurrence of inadequate procedures on Acceleration Comparison Task 1.

<table>
<thead>
<tr>
<th>Procedures</th>
<th>Precourse</th>
<th>Postcourse</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%N^a)</td>
<td>(n^b)</td>
</tr>
<tr>
<td>1. Same slope(^c)</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td>2. Final position</td>
<td>21</td>
<td>(9)</td>
</tr>
<tr>
<td>3. Final speed</td>
<td>38</td>
<td>(16)</td>
</tr>
<tr>
<td>4. Catching up</td>
<td>10</td>
<td>(4)</td>
</tr>
<tr>
<td>5. Greater (\Delta t); same (\Delta x)</td>
<td>12</td>
<td>(5)</td>
</tr>
<tr>
<td>6. Greater (\Delta t); greater (\Delta x)</td>
<td>12</td>
<td>(5)</td>
</tr>
<tr>
<td>7. Same (\Delta x); shorter (\Delta t)</td>
<td>6</td>
<td>(2)</td>
</tr>
<tr>
<td>8. Velocity catches up</td>
<td>100</td>
<td>(2)</td>
</tr>
</tbody>
</table>

\(a\) \(\%N\) = percentage of the total number of occurrences of Procedures 2–8.

\(b\) \(n\) = number of occurrences for each of the Procedures 2–8 used by a selected sample of 35 students.

\(c\) Procedure 1, which is nonkinematical, is not included in the tabulations.

the most important are self-selection of individuals who volunteered to be interviewed, effect of learning on postcourse interviews due to participation in precourse interviews, and intercorser reliability. The first two factors were examined in a manner similar to that described in the previous paper on velocity\(^1\) and found to be negligible. In checking for intercorser reliability an independent investigator, not familiar with either the identity or the course enrollment of the students, listened to 70 tapes of interviews on Acceleration Comparison Task 1 and scored them according to the specified criteria. About 75% of his scores agreed with those assigned by the interviewers and the remainder differed in scoring by no more than one point.

E. Performance on Acceleration Comparison Task 2

Acceleration Comparison Task 2 revealed difficulties with different aspects of the concept of acceleration from those revealed by Acceleration Comparison Task 1. Students successful on one were not necessarily successful on the other. The correct completion of Task 2 requires the ability to distinguish between instantaneous and average velocity. The percentages of each group of students who solved the problem correctly (except for minor arithmetical errors) are shown in Table V and in the graph of Fig. 11.

There was no significant difference in postcourse performance among the three populations (AD, GPL, and CP) to whom Acceleration Comparison Task 2 was administered as a written examination question. In each of these three groups, only about one-fourth of the students were successful. In contrast, about one-half of the students who attempted this task during interviews (GPS) were able to complete it correctly. Guidance from the questioning and lack of time constraints on the interviews probably contributed to this higher rate of success.

V. DISCUSSION OF RESULTS

The results of this investigation indicate that introductory physics students frequently lack even a qualitative understanding of the concept of acceleration as the ratio \(\Delta x/\Delta t\). Performance on Acceleration Comparison Task 1 revealed a widespread inability to apply this concept in a real physical situation. This was the case not only before instruction, but very often afterwards as well. Although most students could define acceleration in an apparently acceptable manner after instruction, they could not use this definition to determine a procedure for comparing the accelerations of two moving objects.

Table V. Results for Acceleration Comparison Task 2. Percentages and numbers (n) of students in each group with correct and incorrect responses.

<table>
<thead>
<tr>
<th></th>
<th>Postcourse interviews</th>
<th>Written examinations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>incorrect</td>
<td>correct</td>
</tr>
<tr>
<td>Academically</td>
<td></td>
<td></td>
</tr>
<tr>
<td>disadvantaged</td>
<td>(AD)</td>
<td></td>
</tr>
<tr>
<td>General physics</td>
<td>(GPS)</td>
<td>52</td>
</tr>
<tr>
<td>(self-paced)</td>
<td></td>
<td>(12)</td>
</tr>
<tr>
<td>General physics</td>
<td>(GPL)</td>
<td></td>
</tr>
<tr>
<td>(lecture)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculus physics</td>
<td>(CP)</td>
<td></td>
</tr>
</tbody>
</table>


D. Trowbridge and L. C. McDermott
The question arises as to whether success on the speed comparison tasks described in an earlier paper was necessary for success on Acceleration Comparison Task 1. In general, those students who successfully resolved this acceleration task had also succeeded on the speed comparison tasks. However, success on the speed tasks did not predict success on the acceleration task. Thus it seems that discrimination between position and velocity was a necessary but not sufficient condition for comparing accelerations.

The poor performance of students on Acceleration Comparison Task 1 could not be attributed to one particular error as was the case with the speed comparison tasks. On the latter we found that failure was almost invariably due to improper use of a position criterion to determine relative velocity. The situation with the concept of acceleration was more complex. From our experience with Acceleration Comparison Task 1, we were able to identify various procedures used by students to compare accelerations.

A. Hierarchy of student understanding

We found that the various procedures used by students in the 200 individual demonstration interviews on Acceleration Comparison Task 1 could be summarized and grouped into ten categories. These could be arranged in a hierarchical order as shown in the chart in Table I. This organization can be interpreted as reflecting different degrees of understanding of the concept of acceleration.

The first four procedures in the chart have a strong perceptual component. We did not consider the use of Procedure 1 as an indication of either the presence or the absence of a conceptual understanding. However, persistence in this nonkinematical approach, after explanation of its unsuitability, was regarded as evidence of the latter. We interpreted the use of Procedures 2-4 as demonstrating a very low level of understanding. Procedure 2, in which position is used as the criterion to compare accelerations, is reminiscent of a similar one used by some students to compare velocities.1

Students invoking Procedure 3 relied upon an intuitive sense of speed and attempted to compare accelerations on the basis of final speeds alone. In Procedure 4 a perception of decreasing separation between the balls, a consequence of relative velocity, is incorrectly attributed to acceleration. Somewhat greater understanding was displayed by students who used Procedures 5-6. These qualitative procedures resembled a comparison of average velocities in that the students considered distances traveled during corresponding time intervals. The tendency to compare accelerations by referring to a criterion based on final velocity, relative velocity, or average velocity indicates a confusion among these concepts.

A greater degree of conceptual understanding is revealed by Procedures 7-8, which require the ability to consider an object in nonuniform motion as having different velocities at different times. Students using these procedures generally could discriminate between velocity and change in velocity. In many instances, Procedure 7 leads to a correct response in the comparison of two accelerations although this procedure fails if the objects have very different velocities. Procedure 8 is valid if the comparison is being made on the basis of a change in relative velocity. In neither of these procedures, however, are the concepts of instantaneous velocity and change in velocity accompanied by explicit reference to a time or a time interval.

Students who used Procedures 9-10 demonstrated that they were able to think of acceleration as the ratio \( \Delta v / \Delta t \), at least in a qualitative sense. By keeping either \( \Delta v \) or \( \Delta t \) constant, they could examine this ratio for the motion of each ball and make a comparison. We interpret success on Acceleration Comparison Task 1 through the use of either of the last two procedures as indicating a qualitative understanding of acceleration as a ratio.

B. Quantitative understanding of acceleration

A task to test for quantitative understanding of the concept as a ratio should involve arbitrary values for the instantaneous velocities and corresponding times. In Acceleration Comparison Task 2, we attempted to explore quantitative aspects of acceleration. An unambiguous interpretation of the results from administration of this task, however, is not possible. The task requires not only a quantitative understanding of the concept of acceleration as a ratio \( (\Delta v / \Delta t) \) but also a clear distinction between instantaneous and average velocity. Furthermore, when the task was presented solely in written form without the intervention of an interviewer, it seemed to encourage almost automatic substitution in memorized formulas. Nevertheless, in spite of these reservations on the interpretation of the results, it is a sobering thought that such a relatively simple problem could cause so much difficulty for students.

C. Persistence of difficulties

The conceptual difficulties with acceleration that were encountered by the students in our study appeared to be very persistent. Often, as illustrated by the pairs of interview excerpts on Acceleration Comparison Task 1, the procedures used by a particular student were the same before and after instruction.

VI. IMPLICATIONS FOR INSTRUCTION

There are a number of implications for instruction that can be drawn from this study. Some of these are discussed in our earlier paper.1 The results from our investigation of student understanding of acceleration reinforce the belief we have already expressed that for many students some form of active intervention is necessary for overcoming confusion between related but different concepts.

One of the most encouraging findings of the study was the great improvement in performance on Acceleration Comparison Task 1 by students from the academically disadvantaged class. Virtually all students in this group began the course unable to discriminate between the concepts of velocity and acceleration. Many also had difficulty distinguishing between position and velocity in comparing two simple motions. After instruction, however, these students demonstrated a qualitative understanding of acceleration as a ratio that matched that of the students from the calculus physics class.

We believe that the type of instruction received by the academically disadvantaged students contributed to this achievement. In their course,2 concept formation received special attention along with the development of scientific reasoning. In laboratory exercises, in class discussions, in individual dialogs with the staff, and on course examinations, the students were confronted with situations designed...
to help them resolve the various kinematical concepts from one another. They were also given practice applying these concepts to the motion of real objects. Like all other participants in the study, however, students in the academically disadvantaged class were presented with Acceleration Comparison Task 1 in pre- and postcourse interviews only.

Implicit in the foregoing remarks is another implication for instruction. The acquisition of concepts of an elementary but fundamental nature often takes an extended period of time. The students in the academically disadvantaged class spent almost a full quarter on kinematics. Although better prepared students may not need that much time, the results of this investigation show that many students from the standard courses were unable to acquire a working understanding of the concept of acceleration in the short time ordinarily allotted to this topic. Perhaps it might be desirable to devote greater attention at the introductory level to the basic kinematical concepts even if this emphasis means that some material on more advanced topics must be omitted.

VII. CONCLUSION

This primarily descriptive study of student understanding of the concept of acceleration has yielded some new insights into how students think about motion. The criterion for understanding used was the ability to apply the concept successfully to the interpretation of simple motions of real objects. Several types of conceptual difficulties were identified.

A significant number of students from a wide variety of courses confused the concepts of velocity and acceleration. Students who succeeded in making the distinction could discriminate between the concepts of instantaneous velocity and change in velocity but often failed to take the corresponding time interval into account. At the completion of instruction, fewer than half of the students demonstrated sufficient qualitative understanding of acceleration as a ratio to be able to apply this concept in a real situation. Even with assistance in making the necessary observations, these students were unable to combine this information in a manner that permitted successful comparison of two accelerations.

This investigation has provided guidance for the design of curricular materials and instructional strategies. We have found the detailed information about conceptual understanding that has emerged particularly useful for helping some college students overcome deficiencies in preparation for introductory physics courses. The results of the study have made possible a concerted effort to address some of the specific difficulties that were identified. By contributing to our knowledge of the ways in which students think about motion, this research has served to suggest new ways in which instruction may be made more effective.

ACKNOWLEDGMENTS

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